Derivadas en coordenadas paramétricas

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1. Determine la ecuación de la recta tangente a la curva:

$$\begin{aligned} x(t) &= -t\cos(t) \\ y(t) &= \sqrt{\pi^2 + 2t^2} \end{aligned} \quad , \quad 0 \le t \le 2\pi$$

en el punto $P = (0, \pi)$

2. Determine la ecuación de la recta normal (perpendicular a la tangente) a la curva

$$C: \begin{cases} x(t) = \sqrt{sen(t^2)} \\ y(t) = \cos^2(t^2) \end{cases} \quad , \quad 0 < t < \frac{\pi}{2}$$

en $t = \dfrac{\sqrt{\pi}}{2}$.

3. Sea $x(t) = 3\cos(t)$; $y(t) = 4sen(t)$, $t \in IR$.

Calcule $\dfrac{d^2y}{dx^2}$ cuando $t = \dfrac{\pi}{3}$

4. Dadas las ecuaciones paramétricas:

$$\begin{aligned} x(t) &= \ln\left(tg\left(\frac{t}{2}\right)\right) + \cos(t) \\ y(t) &= sen(t) \end{aligned} \quad , \quad 0 < t < \pi$$

¿Existe $\dfrac{dy}{dx}$ para todos los valores de $t \in]0, \pi[$?

Para los que exista determine $\dfrac{d^2y}{dx^2}$.

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Solución

1.

$$\frac{dy}{dx}=\frac{y'(t)}{x'(t)}$$

$$\frac{dy}{dx}=\frac{\frac{1}{2\sqrt{\pi^2+t^2}}\cdot 4t}{-\cos(t)-t(-sen(t))}$$

$$\left.\frac{dy}{dx}\right|_{t=0}=0$$

$$y=\pi \quad \Rightarrow \qquad t=0$$

$$y-\pi=0\cdot(x-0)$$

$$y=\pi$$

2. $$x'(t)=\frac{1}{2\sqrt{sen(t^2)}}\cdot\cos(t^2)\cdot 2t$$

$$y'(t)=2\cos(t^2)\cdot(-sen(t^2)\cdot 2t$$

$$\left.\frac{dy}{dx}\right|_{t=\frac{\sqrt{\pi}}{2}}=\left.\frac{y'(t)}{x'(t)}\right|_{t=\frac{\sqrt{\pi}}{2}}=-2\sqrt[4]{2}$$

La pendiente de la recta normal en el punto es $\frac{1}{2\sqrt[4]{2}}$

$$t=\frac{\sqrt{\pi}}{2} \qquad \Rightarrow \qquad x=\frac{\sqrt[4]{2}}{\sqrt{2}} \qquad , \qquad y=\frac{1}{2}$$

$$y-\frac{1}{2}=\frac{1}{2\sqrt[4]{2}}\left(x-\frac{\sqrt[4]{2}}{\sqrt{2}}\right)$$

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3. $x'(t) = -3sen(t) \qquad y'(t) = 4\cos(t)$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$$

$$\frac{dy}{dx} = -\frac{4\cos(t)}{3sen(t)} = -\frac{4}{3}\cot(t)$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{4}{3}\cdot\left(-\cos ec^2(t)\right)}{\frac{dx}{dt}} = \frac{-\frac{4}{3}\cdot\left(-\cos ec^2(t)\right)}{3sen(t)} = -\frac{4}{9sen^3(t)}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{3}} = -\left.\frac{4}{9sen^3(t)}\right|_{t=\frac{\pi}{3}} = -\frac{32}{27\sqrt{3}}$$

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4. $$x'(t)=\frac{1}{tg\left(\frac{t}{2}\right)}\cdot\sec^2\left(\frac{t}{2}\right)\cdot\frac{1}{2}-sen(t)$$

$$x'(t)=\frac{1}{2sen\left(\frac{t}{2}\right)\cos\left(\frac{t}{2}\right)}-sen(t)=\frac{1}{sen(t)}-sen(t)=\frac{\cos^2(t)}{sen(t)}$$

$$y'(t)=\cos(t)$$

$$\frac{dy}{dx}=\frac{y'(t)}{x'(t)}$$

$$\frac{dy}{dx}=\frac{\cos(t)}{\frac{\cos^2(t)}{sen(t)}}=tg(t)$$

Para $0<t<\pi$, $\frac{dy}{dx}$ no existe cuando $\cos(t)=0 \quad\Rightarrow\quad t=\frac{\pi}{2}$

$$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$$\frac{d^2y}{dx^2}=\frac{\sec^2(t)}{\frac{\cos^2(t)}{sen(t)}}=\frac{sen(t)}{\cos^4(t)}\quad \text{para } t\neq\frac{\pi}{2}$$